Gaussian puff formula

$$\langle c(x,y,z,t)\rangle = \frac{q}{2\pi H\sqrt{\bar{K}_{xx}\bar{K}_{yy}}}\exp\left(-\frac{(x-x'-\bar{u}(t-t'))^2}{4\bar{K}_{xx}}-\frac{(y-y')^2}{4\bar{K}_{yy}}\right)$$
$$\times\left\{\frac{1}{2}+\sum_{n=1}^{\infty}\cos\lambda_n z\cos\lambda_n z'\exp[-\lambda_n^2\bar{K}_{zz}]\right\}$$

$$\lambda_n=\frac{n\pi}{H}\qquad \bar{K}_{xx}=\tfrac{1}{2}\sigma_x^2\qquad \bar{K}_{yy}=\tfrac{1}{2}\sigma_y^2\qquad \bar{K}_{zz}=\tfrac{1}{2}\sigma_z^2$$

Total reflection at $z=0$
$\bar{\mathbf{u}}=(\bar{u},0,0)$
$S=q\,\delta(x-x')\,\delta(y-y')\,\delta(z-z')\,\delta(t-t')$
$0\le z\le H$

$$\langle c(x,y,z,t)\rangle = \frac{q}{2\pi\sqrt{\bar{K}_{xx}\bar{K}_{yy}}}\exp\left(-\frac{(x-x'-\bar{u}(t-t'))^2}{4\bar{K}_{xx}}-\frac{(y-y')^2}{4\bar{K}_{yy}}\right)$$
$$\times\sum_{n=1}^{\infty}\frac{(\lambda_n^2+\beta^2)\cos[\lambda_n(H-z')]\cos[\lambda_n(H-z)]}{H(\lambda_n^2+\beta^2)+\beta}$$
$$\times\exp(-\lambda_n^2\bar{K}_{zz})$$

$\lambda_n\tan\lambda_n H=\beta$
$\beta=v_d/K_{zz}$

Partial absorption at $z=0$
$\bar{\mathbf{u}}=(\bar{u},0,0)$
$S=q\,\delta(x-x')\,\delta(y-y')\,\delta(z-z')\,\delta(t-t')$
$0\le z\le H$

Gaussian plume formula

$$\langle c(x,y,z)\rangle = \frac{q}{2\pi\bar{u}\sigma_y\sigma_z}\exp\left(-\frac{y^2}{2\sigma_y^2}\right)\left[\exp\left(-\frac{(z-h)^2}{2\sigma_z^2}\right)+\exp\left(-\frac{(z+h)^2}{2\sigma_z^2}\right)\right]$$

Total reflection at $z=0$
$\bar{\mathbf{u}}=(\bar{u},0,0)$
$S=q\,\delta(x)\,\delta(y)\,\delta(z-h)$
Slender plume approximation
$0\le z\le\infty$

Gaussian plume formula

$$\langle c(x,y,z)\rangle = \frac{q}{2\pi\bar{u}\sigma_y\sigma_z}\exp\left(-\frac{y^2}{2\sigma_y^2}\right)\left[\exp\left(-\frac{(z-h)^2}{2\sigma_z^2}\right)-\exp\left(-\frac{(z+h)^2}{2\sigma_z^2}\right)\right]$$

Total absorption at $z=0$
$\bar{\mathbf{u}}=(\bar{u},0,0)$
$S=q\,\delta(x)\,\delta(y)\,\delta(z-h)$
Slender plume approximation
$0\le z\le\infty$

Gaussian plume formula

$$\langle c(x,y,z)\rangle = \frac{2q}{\sqrt{2\pi}\bar{u}\sigma_y H}\left\{\frac{1}{2}+\sum_{n=1}^{\infty}\cos\left(\frac{n\pi z}{H}\right)\cos\left(\frac{n\pi h}{H}\right)\right.$$
$$\left.\times\exp\left[-\left(\frac{n\pi}{H}\right)^2\frac{\sigma_z^2}{2}\right]\right\}\exp\left(-\frac{y^2}{2\sigma_y^2}\right)$$

Total reflection at $z=0$
$\bar{\mathbf{u}}=(\bar{u},0,0)$
$S=q\,\delta(x)\,\delta(y)\,\delta(z-h)$
$0\le z\le H$
